

Tutorial 5. Oct. 12. 2015.

- Find the principal value of
 - i^i
 - $(1-i)^{4i}$
- Find all values of i^i
- Prove that $\exp(b \log a)$ is single valued if and only if b is an integer.
- $f(z) = z^2$. Let Q be the square with vertices at z , $z+2i$ and $z+2i+2i$. Draw $f(Q)$ and identify the types of image curves correspondingly to the segments from z to $z+2i$ and from $z+2i$ to $z+2i+2i$.

Hint: $z = z + it$ and find $f(z(t))$
 $z = z - t + 2i$ and find $f(z(t))$

5. For each of the following functions, determine all complex numbers for which the function is holomorphic. ~~If you can~~

- (a) \bar{z}^2 (b) $\frac{\sin z}{z^3+1}$ (c) $\text{Log}(z-2i+1)$ (d) $\exp(\bar{z})$
 (e) $(z-3)^i$ (f) i^{z-3}

Hint: (How to find principal value of $\text{Log } z$. $z = x + iy$)

den Step 1. denote $z = x + iy = \sqrt{x^2+y^2} \left(\frac{x}{\sqrt{x^2+y^2}} + i \frac{y}{\sqrt{x^2+y^2}} \right)$

Step 2. find $\alpha, \beta \in \mathbb{R}$ s.t. $\alpha = \log \sqrt{x^2+y^2}$
 (or $\ln \sqrt{x^2+y^2}$)
 $\cos \beta = \frac{x}{\sqrt{x^2+y^2}}$ $\sin \beta = \frac{y}{\sqrt{x^2+y^2}}$

Step 3. Using unique unit circle to find $\beta = \beta_0 + 2k\pi$

Step 4. Find $k_0 \in \mathbb{Z}$ s.t. $\beta_0 + 2k_0\pi \in (-\pi, \pi]$

Step 5. $\text{Log } z$ (Principal value of $\log z$) = $\alpha + (\beta_0 + 2k_0\pi) \cdot i$